Thermodynamics of the self-gravitating ring model: Analyses with new iterative method

Takayuki Tatekawa
Department of Physics, Waseda University, 3-4-1 Okubo, Shinjuku, Tokyo, 169-8555, JAPAN
Advanced Research Institute for Science and Engineering, Waseda University, 3-4-1 Okubo, Shinjuku, Tokyo, 169-8555, JAPAN
Department of Physics, Ochanomizu University, 2-1-1 Otsuka, Bunkyo, 112-8610, JAPAN
E-mail: tatekawa@cosmos.phys.ocha.ac.jp

Freddy Bouchet
Institut Non Linéaire de Nice (INLN) Sophia Antipolis, UMR 6618 CNRS 1361, Allée des Lucioles 06560 Valbonne, FRANCE

Thierry Dauxois
Laboratoire de Physique, UMR-CNRS 5672, ENS Lyon, 46 Allée d’Italie, 69364 Lyon cédex 07, FRANCE

Stefano Ruffo
Dipartimento di Energetica, “S. Stecco” and CSDC, Università di Firenze, and INFN, via S. Marta, 3, 50139 Firenze, ITALY

Abstract. In order to obtain the stable stationary mass distribution, we apply a new iterative method, inspired by a previous one used in 2D turbulence, which ensures entropy increase and, hence, convergence towards an equilibrium state. Applying the new iterative method, we analyze the phase transition and the difference between microcanonical and canonical ensemble in an intermediate energy region.

There are many objects in our universe whose behavior can be understood. Different theoretical approaches have been proposed to explain the peculiar statistical properties of self-gravitating systems. The main difficulty is that these systems cannot approach statistical equilibrium because of the short-distance divergence of the potential and of the evaporation at the boundaries. Even if one puts the system in a box with adiabatic walls, thus eliminating evaporation, still gravity causes the well-known phenomenon of gravothermal catastrophe [1, 2, 3]. The introduction of a small-scale softening of the interaction potential avoids such a catastrophe, so that self-gravitating systems can approach the final (thermal) equilibrium state. However, such a state may have a negative specific heat. Moreover, a first-order phase transition from the high energy gas phase to the low energy clustered phase appears [2].

Recently, an one-dimensional model has been introduced [4] where particle motion is confined on a ring, but the interaction is the true Newtonian 3D one. At short distances, the potential is
Figure 1. Convergence of the entropy for $\varepsilon = 10^{-5}, U = -1$ in the SGR model. The solid line shows the case using new algorithm. During 10 times iteration, the entropy almost converges. The dashed line shows the case using the novel algorithm. After 100 times iteration, the entropy still oscillates and does not converge.

regularized, so that the particles do not interact. This model has been called the Self-Gravitating Ring model (SGR) and will be the subject of the study discussed in this paper. It has been shown in numerical simulations [4], that this model maintains the peculiar features of the 3D Newtonian potential, showing a negative specific heat phase and a phase transition if the softening parameter is small enough. Moreover, for large softening, this model reduces to the Hamiltonian Mean-Field model (HMF) [5], which has been recently extensively studied as a prototype system with long-range interactions. This latter model, however, although it displays a second order phase transition, does not have a negative specific heat phase at equilibrium.

For analyses the equilibrium thermodynamics of the SGR model both in the canonical and in the microcanonical ensemble, we presents a new iterative method which ensures entropy increase and leads in a unique way towards the stable equilibrium single particle distribution function [6]. The method is inspired by a similar one used to compute entropy maxima in 2D turbulence [7].

The distribution at the next step $f_{k+1}$ will be then determined by solving the following variational problem

$$
\max \left\{ S[f] \mid M[f] = 1, E[f_k] + \int \frac{\delta E}{\delta f} \bigg|_{f_k} (f - f_k) \, dp d\theta \leq U \right\}, \quad \frac{\delta E}{\delta f} \bigg|_{f_k} = \frac{p^2}{2} + W_k(\theta).
$$

This variational problem has a unique solution $f_{k+1}$, since it corresponds to the maximization of a strictly concave functional with linear constraints.

For $U < U_c(\varepsilon)$, the stable mass distribution must be determined numerically. We have checked in this case, that a direct iterative method of solution does not always converge. On the contrary, the novel algorithm ensures convergence as shown in Fig. 1 for the entropy.