DESIGN OF A QUASI-OPTICAL SYSTEM CONVERTING THE
TE_{06} OUTPUT MODE OF A GYROTRON INTO A
GAUSSIAN-LIKE BEAM

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Abstract

Gaussian optics can be used to design a quasi-optical system
converting the TE_{06} mode output (f=388 GHz) of a submillimeter wave
gyrotron into a well-collimated, linearly-polarized free-space beam with a
circular cross-section. A quasi-optical antenna produces a main beam
with an elliptical cross-section, which is then converted by two mirrors
into a well-collimated beam with a circular cross-section.

Keywords: Gaussian beam, gyrotron, quasi-optical system
1. Introduction

Electromagnetic waves in the submillimeter wavelength range are used for developments in numerous fields including material physics, plasma diagnostics, astronomy, biophysics, material processing, and so on. Up to now, molecular vapour lasers\(^1\),\(^2\) and backward-wave oscillators\(^3\) have been used as a power source in this wavelength range. Some applications like plasma scattering measurements\(^4\)-\(^6\) need higher output powers. High frequency gyrotrons offer clear advantages in this wavelength range due to their frequency step tunability and high output powers from several hundred watts up to several kW\(^7\)-\(^9\).

To be used as a convenient radiation power source, the gyrotron output should be converted into a well-collimated, linearly-polarized beam with a circular cross-section. A field pattern of the gyrotron output differs significantly from usual specifications of an experiment.

We have proposed a quasi-optical system converting the TE\(_{15}\) mode output (\(f=354\) GHz) of a submillimeter wave gyrotron into a high quality beam\(^10\). The system consists of a quasi-optical antenna\(^11\) and an elliptical mirror. The antenna converts the gyrotron output into a linearly-polarized beam containing not only the main beam with a circular cross-section, but also additional sidelobes. In order to improve the beam quality, the sidelobes are truncated by reducing the size of the elliptical mirror located far from the antenna\(^12\).

However, the design described above is not applicable to a TE\(_{0n}\) mode output. Since this type of antenna produces a main beam with an elliptical cross-section in the far field region. As can be seen from Gaussian optics, such beam can be converted into a circular one by two-dimensionally adjusting the spotsize and the curvature radius of the wave front of the beam.

As an example of the approach, we will design a quasi-optical system for the TE\(_{06}\) mode output (\(f=388\) GHz) of the Gyrotron FU II\(^13\) employing the quasi-optical antenna optimized for TE\(_{15}\) mode output (\(f=354\) GHz) with only two additional correcting mirrors. Such an approach is important for the compatibility of the system with the frequency tunability of the gyrotron. Since it allows us to apply the antenna to both TE\(_{0n}\) and TE\(_{1n}\) modes.

Because in our design, the distance between the antenna and the following mirror is long, the Gaussian optics can be applied to the beam after the antenna. This treatment is effective to decrease the required
2. Treatment of the beam using Gaussian optics

The first element in the quasi-optical system is the quasi-optical antenna (Fig.1) consisting of a circular waveguide (internal radius $a_w = 14$ mm) with a step-cut and a parabolic reflector (focal length $f_p = 21.75$ mm). The antenna converts the gyrotron output into a linearly-polarized beam. Its electric and magnetic fields are parallel to the $x$- and $y$-directions, respectively.

Radiation reflected from the parabolic reflector of the quasi-optical antenna is treated as if it comes from a plane image source\(^{14,15}\) lying behind the parabolic reflector with the emission angle $\alpha$. To design the quasi-optical system (Fig.2), the image source is located in such that the beam with polarization in the $x$-direction propagates along the $z$-axis.

Fig.1 Quasi-optical antenna. The $x$- and $y$-directions correspond to the directions of electric and magnetic fields, respectively.
Fig. 2 Quasi-optical system. A plane image source is used for the calculation of the subsequent radiation patterns of the quasi-optical antenna with a single parabolic reflector. The beam propagates along the $z$-axis.

Other components of this system are the mirrors $m_1$ and $m_2$. The distance between them is 2000 mm. The distance between image source and the mirror $m_1$ (7000 mm) is long enough to apply Gaussian optics.

In our previous paper\textsuperscript{12}), we have shown that such an antenna produces the main beam with a circular cross-section in the far field region, when Gyrotron FU II produces the $\text{TE}_{15}$ mode output ($f=354$ GHz, emission angle $\alpha =8.23^\circ$). If we apply it to the $\text{TE}_{06}$ mode output ($f=388$ GHz, $\alpha =9.97^\circ$), the main beam produced has an elliptical cross-section, as can be seen from the intensity profile at the mirror $m_1$ (Fig.3). In order to improve the quality of the beam by truncating the sidelobes, the size of the rectangular mirror $m_1$ is determined as 144 mm in the $x$-direction and 195 mm in the $y$-direction. On the other hand, the mirror $m_2$ is wide enough to avoid any diffraction loss due to the beam truncation.

The spot sizes of the main beam for larger distances from the image source are accurately calculated employing a bigaussian beam with the waist (the size $w_{0x}=38.7$ mm in the $x$-direction, $w_{0y}=25.2$ mm in the $y$-direction) located at the center of the image source. In the present case, the intensity of the bigaussian beam is given by
Fig. 3  Calculated intensity contours at the mirror m1. Contours are in
decibels relative to the maximum intensity.

\[
I = \frac{2P_0}{\pi \omega_x \omega_y} \exp\left(-\frac{2x^2}{\omega_x^2}\right)\exp\left(-\frac{2y^2}{\omega_y^2}\right),
\]

(1)

where \(\omega_x\) and \(\omega_y\) are the spot sizes of the bigaussian beam in the \(x\)- and \(y\)-
directions, respectively and \(P_0\) is the total beam power.

The complex beam parameters \(q_x\) and \(q_y\) in the \(x\)- and \(y\)-directions
are convenient parameters to treat the beam propagation as well as its
focusing by the mirror elements. They are defined by

\[
\frac{1}{q_x} = \frac{1}{R_x} - j \frac{\lambda}{\pi \omega_x^2}
\]

\[
\frac{1}{q_y} = \frac{1}{R_y} - j \frac{\lambda}{\pi \omega_y^2}
\]

(2)

where \(R_x\) and \(R_y\) are the curvature radii of the wave fronts in the \(x\)- and \(y\)-
directions, respectively.

At the beam waist, \(R_x, R_y \rightarrow \infty\). Therefore, the complex beam
parameters are

\[ q_{0x} = j \frac{\pi w_{0x}^2}{\lambda} \]

\[ q_{0y} = j \frac{\pi w_{0y}^2}{\lambda} \quad (3) \]

When the beam propagates or is focused by any element, the complex beam parameters change. After propagating a distance \( d \), the complex beam parameters \( q_x \) and \( q_y \) change to new values \( q_{x'} \) and \( q_{y'} \) given by

\[ q_{x'} = q_x + i \]

\[ q_{y'} = q_y + d \quad (4) \]

After the focusing element, they change to new values \( q_{xf} \) and \( q_{yf} \) given by

\[ \frac{1}{q_{xf}} = \frac{1}{q_x} - \frac{1}{f_x} \]

\[ \frac{1}{q_{yf}} = \frac{1}{q_y} - \frac{1}{f_y} \quad (5) \]

where \( f_x \) and \( f_y \) are the focal lengths of the focusing element in both directions.

3. Design of the system using Gaussian optics

To demonstrate the method, we have designed a quasi-optical system (Fig.2) to produce a well-collimated beam with a circular cross section (the waist size \( w_{0x} = w_{0y} = 5 \) mm).

The design is accomplished by converting the complex beam parameters \( q_{0x} \) and \( q_{0y} \) (\( q_{0x} \neq q_{0y} \)) at the image source into the two-dimensionally equal complex beam parameter \( q_{0x'} \) and \( q_{0y'} \) (\( q_{0x'} = q_{0y'} \)) at the beam waist according to eqs. (4) and (5). The values of \( q_{0x} \) and \( q_{0y} \) are obtained using eq. (3) with \( w_{0x} = 38.7 \) mm and \( w_{0y} = 25.2 \) mm. The values of \( q_{0x'} \) and \( q_{0y'} \) are obtained from the same eq. (3) with \( w_{0x} = w_{0y} = 5 \) mm.

As can be seen from eqs. (4) and (5), we need two steps (this means, focusing and propagation) to equalize the spot size in the x-
direction with that in y-direction at the mirror m2. Hence, the mirror m1 helps to equalize both spot sizes at the mirror m2. As follows from eq. (2), this is expressed by

$$\text{Im} \left( \frac{1}{q_{2x}} \right) = \text{Im} \left( \frac{1}{q_{2y}} \right),$$

(6)

where $q_{2x}$ and $q_{2y}$ are the complex beam parameters in the z- and y-directions at the mirror m2. Because the direction of the beam propagation changes from the x-direction to the z-direction after the beam reflection at the mirror m1, the subscript 2x of the complex beam parameter $q$ is replaced by 2y.

The cross-section in the y-direction is wider than that in the x-direction at the mirror m1 (Fig. 3). For simplicity, we assume that the mirror m1 has only to focus the beam in the y-direction. Correspondingly, we use there

$$f_{1y} = \infty,$$

(7)

where $f_{1y}$ is the focal length of the mirror m1. If the distance between the mirrors m1 and m2 is given ($d_{12} = 2000$ mm), the focal length $f_{1y}$ of the mirror m1 is given by eq. (6), so $q_{2x}$ and $q_{2y}$ can be determined as well.

As can be seen from eq. (5), one step is enough to equalize the curvature radii of the wave front in the z-direction (x-direction) and in the y-direction. This is done by the mirror m2. The condition is expressed by

$$\frac{1}{q_{2x}} = \frac{1}{f_{2x}} = \frac{1}{q_{2y}} \times \frac{1}{f_{2y}} = \frac{1}{q_{2y}'}.$$

(8)

where $f_{2x}$ and $f_{2y}$ are the focal lengths of the mirror m2 and $q_{2x}$, and $q_{2y}$, ($q_{2x}'=q_{2y}'$) are the complex beam parameter just after reflection by the mirror m2. Since, according to eq. (4), the imaginary part of the complex beam parameter does not change along the beam propagation, we derive the condition

$$\text{Im} \left( q_{2x}' \right) = q_{0x}.$$

(9)

The values of $f_{2x}$ and $f_{2y}$ are also given by eqs. (8) and (9), hence the distance $d_{2w}$ between the mirror m2 and the beam waist can be determined. The results obtained in such a way by Gaussian optics are listed in Table 1.

4. Mirrors m1 and m2

The mirror m1 is a parabolic cylinder whose focal axis is located in
the x-z plane (Fig.4). If the rays coming from the axis F₁ propagate along the z-direction and are incident on the mirror, the reflected rays are focused in the point F₂. In our case, the focal length \( f₁y = \frac{f_p}{\cos 45°} = 6497 \text{ mm} \), where \( f_p = 4594 \text{ mm} \) is the focal length of the parabolic cylinder.

The shape of the mirror m2 is derived from the equation of an ellipse. In the case when the focal points F₁ and F₂ and the mirror center \( P_c \) are located in the \( y=0 \) plane (Fig.5), the equation is given by

Table 1. The results obtained by Gaussian optics and those by calculation. \( d_{2ox} \) and \( d_{2oy} \) denote the distances between the center of the mirror m2 and beam waist.

<table>
<thead>
<tr>
<th></th>
<th>Mirror m1 ( f₁x = \infty )</th>
<th>Mirror m2 ( f₂x = 1271 )</th>
<th>Beam waist</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( w_x ) (mm)</td>
<td>( w_y ) (mm)</td>
<td>( w_z ) (mm)</td>
</tr>
<tr>
<td>Gaussian optics</td>
<td>59.0</td>
<td>72.9</td>
<td>69.1</td>
</tr>
<tr>
<td>Calculation</td>
<td>48.8</td>
<td>68.8</td>
<td>58.4</td>
</tr>
</tbody>
</table>

Fig.4 Geometry of the mirror m1.
Fig. 5 Geometry of the mirror m2.

\[ g(x,y,z) = d_1 + d_2 = d_{1c} + d_{2c} \]
\[ d_1 = \sqrt{(x-x_1)^2 + y^2 + (z-z_1)^2} \]
\[ d_2 = \sqrt{(x-x_2)^2 + y^2 + (z-z_2)^2} \]
\[ d_{1c} = \sqrt{(x_c-x_1)^2 + (y_c-y_1)^2} \]
\[ d_{2c} = \sqrt{(x_c-x_2)^2 + (y_c-y_2)^2} \]

where \( d_1 \) is the distance between the focal point \( F_1(x_1,0,z_1) \) and an arbitrary point \( P(x,y,z) \) on the surface, \( d_2 \) is measured between the other focal point \( F_2(x_2,0,z_2) \) and the same point \( P(x,y,z) \) and \( d_{1c} \) and \( d_{2c} \) are distances between the focal points and the mirror center \( P_c(x_c,0,z_c) \), respectively. This ellipsoidal mirror focuses in the second focal point \( F_2 \) the rays originating from the focal point \( F_1 \). The focal length \( f \) of the mirror is given by

\[ \frac{1}{f} = \frac{1}{d_{1c}} + \frac{1}{d_{2c}} \]  

The shape of the mirror m2 is deduced by modifying eq. (10) into

\[ g(x,y,z) = d_1' + d_2 = d_{1c} + d_{2c} \]
where
\[ d_1' = \sqrt{(x-x_1)^2 + (z-z_1)^2}. \]

The modified mirror focuses in the focal point \( F_2 \) the rays, which are parallel to the \( y=0 \) plane and originate from the axis \((x_1,y,z_1)\). In order to show this, let us start with the derivation of the normal vector at the point \( P \). Since any small displacement \( \mathbf{dr} \) along the surface does not change the value of \( g(x,y,z) \), one gets
\[ \mathbf{dg} = (\nabla g) \cdot \mathbf{dr} = 0. \quad (13) \]

It shows that the vector \( \nabla g \) is perpendicular to \( \mathbf{dr} \), and thus, it is parallel to the normal vector. Using eqs. (12) and (13), \( \nabla g \) is calculated as
\[ \nabla g = \left( \frac{x-x_1}{d_1} + \frac{x-x_2}{d_2} \right) \mathbf{i} + \frac{y}{d_2} \mathbf{j} + \left( \frac{z-z_1}{d_1} + \frac{z-z_2}{d_2} \right) \mathbf{k}, \quad (14) \]

where \( \mathbf{i}, \mathbf{j} \) and \( \mathbf{k} \) denote the unit vectors along the \( x, y \) and \( z \)-directions. Therefore, \( \nabla g \) is the sum of the unit vector parallel to the \( y=0 \) plane from the focal axis to the point \( P \) and the unit vector in the direction from the focal point \( F_2 \) to the point \( P \).

According to the law of reflection, it is obvious that the mirror focuses in the focal point \( F_2 \) the rays, which are parallel to the \( y=0 \) plane and originate from the axis. The focal length \( f_{2y} \) in the \( y \)-direction of the mirror is different from \( f_{2x} \) in the direction perpendicular to the \( y \)-direction. As can be seen from the cases of mirror \( \mathbf{m}_1 \) and the ellipsoidal mirror, they are given by
\[ \frac{1}{f_{2x}} = \frac{1}{d_{1c}} + \frac{1}{d_{2c}} \]
\[ \frac{1}{f_{2y}} = \frac{1}{d_{2c}}. \quad (15) \]

If we select \( d_{1c}=9900 \text{ mm}, d_{2c}=1458 \text{ mm}, \) the mirror satisfies the conditions \((f_{2x}=1271 \text{ mm}, f_{2y}=1458 \text{ mm})\) obtained from Gaussian optics. The focal points and the mirror center fitting into the system shown in Fig.2 are \( F_1 (-7900,0,7000), F_2 (2000,0,8458) \) and \( P_c(2000,0,7000) \).

5. Verification using Huygens' equation

In order to verify the results obtained by Gaussian optics, we have calculated the beam profiles using the Huygens' equation. First, the incident electromagnetic fields at the surface of the mirror \( \mathbf{m}_1 \) are
calculated. The electromagnetic fields reflected from the mirror are obtained using the boundary condition for a perfect conductor. Second, the electromagnetic fields on the subsequent mirror m2 are obtained by repeatedly using the Huygens equation together with the boundary condition and taking the previously calculated results as sources.

Calculated intensity contours in the vicinity of the mirror m2 are shown in Fig.6. The beam spreads out in the z-direction, but it converges in the y-direction due to the mirror m1 and it approaches an almost circular cross-section at the mirror m2 (Fig.7).

The beam produced by the image source contains sidelobes in addition to the main beam, as can be seen in Fig.3. The quality of the beam is improved by truncating the sidelobes, i.e. by limiting the size of the mirror m1 to the optimal one. This attempt appears effective in removing sidelobes (Fig.7). In spite of the truncation of the sidelobes, most of the power from the image source (81.6 %) is still reflected by the mirror m1.

Calculated intensity contours in the vicinity of the beam waist are shown in Fig.8. This beam has almost the same profile in both x- and y-directions. Calculated intensity contours at the beam waist are shown in Fig.9. The spot sizes, the waist sizes and their positions obtained from the

![Fig.6](image)

Fig.6 Calculated intensity contours in the vicinity of the mirror m2, in (a) the x-z plane and in (b) the x-y plane. Contours are relative to the intensity along the x-axis.
calculations are also listed in Table 1 for comparison. The results obtained by the Gaussian optics approach are in good agreement with these calculations.

6. Conclusion

Calculations using the Huygens equation confirm that the propagation of the beam produced by the image source can be treated far away from the source as a bigaussian beam. Although the quasi-optical antenna produces the main beam with an elliptical cross-section in the far-field if its feeding differs from its optimized mode (TE_{15} mode in our case), the quasi-optical system designed by Gaussian optics is able to convert the TE_{06} mode output (f=388 GHz) from a submillimeter wave gyrotron into a well-collimated, linearly-polarized beam with a circular cross-section. This allows us to apply the antenna to both TE_{0n} and TE_{1n} modes.

![Fig. 7] Calculated intensity contours at the mirror m2. Contours are in decibels relative to the maximum intensity.
Fig. 8 Calculated intensity contours in the vicinity of the beam waist. Contours are relative to the intensity along the z-axis.
Fig. 9 Calculated intensity contours at the beam waist.

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