Quick estimation of mode content in a submillimeter-wave gyrotron output

M. Pereyaslavets, T. Idehara, I. Ogawa, and S. Mitsudo

1Department of Applied Physics
Faculty of Engineering
Fukui University
Fukui 910-8507, Japan
2Research Center for Development of Far-Infrared Region
Fukui University
Fukui 910-8507, Japan
3Cryogenic Laboratory
Faculty of Engineering
Fukui University
Fukui 910-8507, Japan

Abstract
An inexpensive and quick method for the approximate estimation of the mode content of the output of a submillimeter-wave gyrotron requires a thermographic pattern measurement in one cross section only.

Keywords: Mode Content, Gyrotron, Submillimeter-Wave

1. Introduction

Medium power submillimeter-wave gyrotrons have been developed as frequency tunable sources covering the whole submillimeter-wave range. They become more and more important in the far-infrared region [1]. Due to various reasons, gyrotron outputs contain not only the operating waveguide
mode, but also some parasitic modes. The power in the parasitic modes may suddenly increase even in a steady operating gyrotron. Therefore, it is necessary to analyze the mode content of a gyrotron output from time to time.

Mathematically this problem can be reduced to retrieval of field phase from intensity measurements. Effective algorithms have been developed for that purpose in [2], [3], and they were successfully applied to field profile synthesis of gyrotron output radiation [4], [5], [6]. Similar techniques are known in other applications [7] - [13].

Unfortunately, all those methods require high-resolution field intensity measurements in two or three different cross-sections. Hence, they employ an infrared or thermographic camera [9], [12], [14], [15], [16] or antenna arrays [6], [11], [17] or scanning probes [10], [18] or horns [19], or SAW spectrometers with far field RF scans [20], or k-spectrometers for mode content analysis [21], [22].

Some effort has been devoted to decreasing the number of sampling measurements [7], [8]. Nevertheless, all such measurements still require expensive and sophisticated experimental equipment and are also time consuming. As it was pointed out in [7], one intensity measurement only can be sufficient for field reconstruction, if some constraints are added.

In the case of our gyrotrons, one usually knows what the main parasitic modes are and the problem can be reduced to determination of parasitic mode powers. In this paper we have employed very quick and inexpensive measurements of field intensity by a liquid crystal sheet placed in front of the gyrotron output.

This intensity measurement by thermographic pattern analyzers is a well-known technique in waveguides and gyrotrons. For example, it is described in [23]. Similar measurements were carried out in [24] for mode analysis in
several cross sections. Whereas such a pattern analysis is quick and inexpensive, its spatial resolution is extremely low. Moreover, a liquid crystal film has a very non-linear response: high intensity areas on such a film are almost ‘burned out’ after a few seconds of exposure [25]. One can only determine relative intensities for several different areas of the pattern, but the spatial resolution is not sufficient to warrant intensity measurements in more than one cross-section and application of the algorithm [2].

2. Retrieval method.

Let us assume the gyrotron output contains $N$ circular waveguide modes, hence the electrical field is

$$
\vec{E}(r,\varphi) = \sum_{m=1}^{N} A_m \exp(i\Phi_m) \vec{e}_m(r,\varphi);
$$  \hspace{1cm} (1)

where $\vec{e}_m$ is the normalized electrical field of the $m$-th mode, $A_m$ and $\Phi_m$ are its amplitude and phase, respectively; and the polar coordinates are $(r,\varphi)$. Since we know which modes are contained in eq. (1), our problem is to determine relative mode amplitudes, phases and powers. The measured intensity at each point is proportional to $|\vec{E}|^2$, i.e.

$$
P(r,\varphi) = |\vec{E}(r,\varphi)|^2. \hspace{1cm} (2)
$$

As it was mentioned in the Introduction, the spatial resolution of a thermographic pattern is low, hence one can obtain only a few of relative intensities. This means one can measure, for
example, the relative intensity of two rings contained between \( r_1 \), \( r_2 \) and \( r_3, r_4 \), yielding:

\[
\frac{\iint_0^{2\pi} P(r,\varphi) \, d\varphi \, dr}{\iint_0^{2\pi} |\mathbf{E}(r,\varphi)|^2 \, d\varphi \, dr} = \left( \frac{\iint_0^{2\pi} |\mathbf{E}(r,\varphi)|^2 \, d\varphi \, dr}{\iint_0^{2\pi} P(r,\varphi) \, d\varphi \, dr} \right)^2 \tag{3}
\]

In a similar way, the relative intensity of two sectors of a circle of the radius \( a \) contained between \( \varphi_1 \), \( \varphi_2 \) and \( \varphi_3 \), \( \varphi_4 \), respectively can be determined as:

\[
\frac{\iint_{\varphi_1}^{\varphi_2} P(r,\varphi) \, dr \, d\varphi}{\iint_{\varphi_1}^{\varphi_2} |\mathbf{E}(r,\varphi)|^2 \, dr \, d\varphi} = \left( \frac{\iint_{\varphi_1}^{\varphi_2} |\mathbf{E}(r,\varphi)|^2 \, dr \, d\varphi}{\iint_{\varphi_1}^{\varphi_2} P(r,\varphi) \, dr \, d\varphi} \right)^2 . \tag{4}
\]

Depending on a specific thermographic pattern, one must substitute eq. (1) into the corresponding eqs. (3) and (4) and determine relative intensities of several rings and/or sectors to derive \( 2(N - 1) \) simultaneous equations. Solving those equations, one yields relative amplitudes and phases of \( N \) modes in eq. (1). Thereafter one would employ eq. (1) and reconstruct the thermographic pattern. If the initial assumption on the contributing parasiting modes in eq. (1) is correct, the reconstructed thermographic pattern would be very similar to the measured one.

3. Practical examples.

To illustrate the method described above we have employed
GYROTRON FU IV A [1]. Thermographic patterns were obtained by placing a crystal sheet in front of the gyrotron output window for several seconds. The measurements were carried out in an initial stage of gyrotron installation, where the level of parasitic modes was relatively high. Fig. 1 presents the thermographic patterns for the dominant TE\(_{23}\) mode for exposures of 5 seconds, 6 seconds, and 7 seconds, respectively. The frequency was 294.7 GHz, the main magnetic field 10.53 T,

![Thermographic patterns for different exposures](image)

**Fig. 1.** Measured patterns for the TE\(_{23}\) dominant mode.

output power 11 W, the cathode voltage 14 kV and the beam current 116 mA.

It is reasonable to assume the main parasitic mode in Fig. 1 is TE\(_{22}\). To estimate its power and phase, we have compared intensities of three rings, thus deriving two simultaneous equations similar to eq. (3). Solving those equations, we yield the estimation of the relative powers of the TE\(_{23}\) and TE\(_{22}\) modes as 91\%-94\% and 6\%-9\%, respectively. The phase difference between the modes is estimated as \(\pi\). The reconstructed pattern for the relative powers TE\(_{23}\) of 93\%, and TE\(_{22}\) of 7\% with the phase difference \(\pi\) is presented in Fig. 2.

Of course, a more sophisticated experimental technique like [4] or [6] with an IR camera would produce an essentially better
reconstruction. Nevertheless, taking into account simplicity of our

Fig. 2. Reconstructed pattern; 93% TE$_{23}$, 7% TE$_{22}$, phase $\pi$.

equipment, the pattern of Fig. 2 is a reasonable approximation.

Another example is shown in Fig. 3. Those thermographic patterns are for the dominant TE$_{02}$ mode for the exposures of 3, 4, 5 and 6 seconds, respectively. The frequency was 207.5 GHz, the main magnetic field 7.42 T, output power 2.2W, the cathode voltage 15 kV and the beam current 60 mA.

Fig. 3. Measured patterns for the TE$_{02}$ dominant mode.

Here we assume the TE$_{22}$ parasitic mode and compare intensities of two rings producing eq. (3). The second simultaneous equation is derived by comparing intensities of two
sectors in Fig. 3 and is similar to eq. (4). The solution of those simultaneous equations yields the estimation of the relative powers of the TE\(_{02}\) and TE\(_{22}\) modes as 85%-87% and 13%-15%, respectively. The phase difference between the modes is estimated as 0. The reconstructed pattern for the relative powers TE\(_{02}\) of 86%, and TE\(_{22}\) of 14% with the phase difference 0 is presented in Fig. 4. It is in reasonable agreement with the measured one. Let us note that the measured pattern in Fig. 3 suggests also a contribution from the TE\(_{13}\) parasitic mode in addition to TE\(_{22}\). However, the spatial resolution in Fig. 3 does not allow us to determine the level of TE\(_{13}\).

![Image](image.png)

Fig. 4. Reconstructed pattern; 86% TE\(_{02}\), 14% TE\(_{22}\), phase 0.

5. Conclusion.

An inexpensive and quick method for approximate estimation of mode content was applied to a submillimeter-wave gyrotron. The method analyses a thermographic pattern measured in only one cross section. Such a technique can be useful to spare measurement time and sophisticated equipment in various frequency bands. It is especially reasonable in submillimeter-wave region.
Acknowledgement

This work was supported in part by a grant-in-aid from the Ministry of Education, Science, Sport and Culture of Japan. Thanks are due to Prof. Dr. M. Thumm from the Forschungszentrum Karlsruhe and Dr. G. F. Brand from the University of Sydney for invaluable discussions.

References

[2] Katsenelenbaum B.Z. and Semenov V.V. Synthesis of phase correctors shaping a specified field, Radio Engineering and Electronic Phys, no. 12, 244-252 (1967)