MODELLING AND SIMULATION OF MAGNETRON INJECTION GUNS FOR SUBMILLIMETER WAVE GYROTRONS

S. P. Sabchevski,1 G. M. Mladenov,1 and T. Idehara2

1Institute of Electronics
Bulgarian Academy of Sciences
72 Tzarigradsko Shose Boulevard
Sofia 1784, Bulgaria
2Faculty of Engineering
Fukui University
3-9-1 Bunkyo
Fukui-shi 910-8507, Japan

Abstract

The software package GUN-MIG has been developed for computer simulation of beam formation in magnetron injection guns (MIG). It is based on a fully relativistic self-consistent physical model which takes into account the specific problems and requirements that are being encountered in the analysis and design of MIG for gyrotrons. Besides providing a general outline of the physical models and the program implementation of the code, several illustrative examples of the numerical experiments with real MIG for submillimeter wave gyrotrons are presented and discussed.

Key words: Gyrotrop, magnetron injection gun, trajectory analysis
1. Introduction

Intense electromagnetic waves in the millimeter and submillimeter range are widely used in the basic and applied physical research, advanced radar and telecommunication systems as well as in the materials processing technology. A comparatively novel class of microwave devices generating radiation in this region of the electromagnetic spectrum is based upon the cyclotron resonance maser instability which originates from the electron azimuthal bunching due to the dependence of electron relativistic gyration frequency on energy [1]. Among them, the gyrotron has emerged as the most promising radiation source for the development of the far-infrared region. There are two distinct directions in which the development of gyrotrons is proceeding. One is the development of high power, millimeter wave gyrotrons as power sources for electron cyclotron heating of fusion plasmas in tokamaks and for ceramics sintering [2,3]. The other is the development of high frequency, medium power gyrotrons for plasma diagnostics and far-infrared spectroscopy [4,5]. The gyrotrons developed in the Fukui University (Gyrotron FU Series) belong to the latter type and cover a wide frequency range in the millimeter to submillimeter wavelength. The submillimeter wave gyrotrons have many advantages, including frequency tunability, moderately high power, high stability of amplitude and frequency, effective modulation of the amplitude and frequency, high purity mode operation. Operation at harmonics of the electron cyclotron frequency makes it possible to overcome the upper frequency limit set by the available superconducting magnets and, consequently, to reduce the weight and size of the devices.

The gyrotron tube consists of the following main parts: (i) Magnetron injection gun (MIG) which forms an annular helical electron beam; (ii) Beam drifting region in which the beam is compressed by a gradually increasing magnetic field and where the transverse energy of the electrons increases at the expense of their axial energy; (iii) Resonant cavity in which the electron beam interacts with and excites a high frequency microwave field; and (iv) Electron beam collector which also serves as a part of an output waveguide. After the interaction with the corresponding mode, the spent beam is allowed to decompress and is deposited on the collecting surface.
Although the MIG represents only a small fraction of the gyrotron, its performance is crucial for the over-all operational characteristics of the entire device. Computer simulation is a powerful means for analysis, optimization and computer assisted design (CAD) of such guns [6-8]. The advantage of simulation (numerical experiments) is enormous as it replaces many complicated, laborious, expensive and time consuming direct experiments and allows to explore a very large number of alternative geometric configurations of the electrodes and external fields, looking for the optimal one. In this respect, the development of appropriate software tools for CAD, based on adequate physical models and efficient numerical methods and algorithms is an indispensable aspect of the engineering of high-performance MIGs.

It is the intent of this paper to outline the current version of the software package GUN-MIG specifically designed for analysis and optimization of MIGs. It is based on a self-consistent physical model in which both the relativistic and the space-charge effects are taken into account. For calculation of the electrostatic potential distribution in the gun and extracted beam current the methods, techniques and software modules realized in the GUN-EBT software package [9] are used. In contrast to the GUN-EBT however, the space-charge distribution is computed by the particle-in-cell (PIC) method. In its present form, GUN-MIG traces electron orbits in combined electric and magnetic fields and can be used for trajectory analysis of beams generated in axially symmetric MIGs.

2. Physical model and software

Basic assumptions of the self-consistent physical model which describes beam formation in the gun are: (i) The electron gun and the electromagnetic field are axisymmetric. (ii) The electromagnetic field is static and the beam is time independent (steady-state conditions). For MIGs that generate pulsed beams we consider pulse lengths long compared to the transit time across the gun and do not treat transient phenomena. (iii) The magnetic field induced due to beam current is neglected.
In a cylindrical coordinate system \((R, \theta, Z)\) the relativistic equations of motion can be derived from the Lorentz force equation in the following form [10]:

\[
\ddot{R} = \frac{1}{m_0} (1 - \beta^2) \left[ f_R - \frac{e}{c^2} \dot{R}(\dot{R}E_R + \dot{R}\dot{\theta}E_\theta + \dot{Z}E_Z) \right] + R\ddot{\theta}^2, \tag{1}
\]

\[
\ddot{\theta} = \frac{1}{m_0 R} (1 - \beta^2) \left[ f_\theta - \frac{e}{c^2} \dot{R}(\dot{R}E_R + \dot{R}\dot{\theta}E_\theta + \dot{Z}E_Z) \right] - \frac{2\dot{R}\dot{\theta}}{R}, \tag{2}
\]

\[
\ddot{Z} = \frac{1}{m_0} (1 - \beta^2) \left[ f_Z - \frac{e}{c^2} \dot{R}(\dot{R}E_R + \dot{R}\dot{\theta}E_\theta + \dot{Z}E_Z) \right], \tag{3}
\]

where \(m_0\) is the electron rest mass, \(c\) is the speed of light, \(f_R = eE_R + e(\dot{R}\dot{\theta}B_\theta - \dot{Z}B_\phi), f_\theta = eE_\theta + e(\dot{Z}B_\phi - \dot{R}B_\theta), f_Z = eE_Z + e(\dot{R}B_\theta - \dot{\theta}B_R), \beta^2 = (\dot{R}^2 + (\dot{\theta}^2 + \dot{Z}^2))/c^2\) and \(E, B\) are electrostatic and magnetic fields. In the present implementation of the program only the external magnetic field is taken into account while the self-magnetic field of the beam \(B_0\) is neglected. From the azimuthal symmetry, \(\phi\) do not depend on the azimuthal angle \(\theta\), i.e. \(\frac{\partial \phi}{\partial \theta} = 0\) and hence \(E_\theta = 0\). The axial and the radial components of the electric field \((E_z = \frac{\partial \phi}{\partial Z}, E_R = \frac{\partial \phi}{\partial R})\) can be calculated from the electrostatic potential distribution, which obeys the Poisson's equation

\[
\frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \phi}{\partial R} \right) + \frac{\partial^2 \phi}{\partial Z^2} = -\frac{\rho}{\varepsilon_0}, \tag{4}
\]

as well as appropriate Dirichlet and Neumann boundary conditions. Here \(\rho\) is the space charge density and \(\varepsilon_0\) is the dielectric constant of free space. A square-shaped grid and a five-point finite difference discretization formula are used for the solution of the boundary value problem. The potential in an arbitrary point \((R, Z)\) is approximated by the Lagrange interpolation in the form [11]

\[
\phi(R, Z) = \sum_{i=1}^{4} \sum_{j=1}^{4} a_i b_j \phi_{ij}, \tag{5}
\]

where
\[ a_i(Z) = \left[ \prod_{k=1}^{4} (Z_i - Z_k) \right]^{-1} \prod_{k=1}^{4} (Z - Z_k), \]  
(6)

\[ b_j(R) = \left[ \prod_{k=1}^{4} (R_j - R_k) \right]^{-1} \prod_{k=1}^{4} (R - R_k) \]  
(7)

and \( \phi_j \) are the potentials at the adjacent mesh points with coordinates \( R_j, Z_i \). Accordingly, the electric field components at the same points are given by

\[ E_x = -\sum_{i=1}^{4} \sum_{j=1}^{4} a_i \frac{db_j}{dR} \phi_j, \quad E_z = -\sum_{i=1}^{4} \sum_{j=1}^{4} b_j \frac{da_i}{dZ} \phi_i. \]  
(8)

The axial magnetic field is fitted by a sixth order polynomial expansion

\[ B(0, Z) = B = \sum_{i=1}^{7} B_i Z^{i-1} \]  
(9)

The off-axis components of the magnetic field are computed from the following expansions [12]

\[ B_x(R, Z) = B - \frac{R^2}{4} \left( \frac{d^2 B}{dZ^2} - \frac{R^2}{16} \frac{d^4 B}{dZ^4} + \frac{R^4}{576} \frac{d^6 B}{dZ^6} \right), \]  
(10)

\[ B_y(R, Z) = -\frac{R}{2} \left( \frac{d B}{dZ} - \frac{R^2}{8} \frac{d^3 B}{dZ^3} + \frac{R^4}{192} \frac{d^5 B}{dZ^5} \right). \]  
(11)

The space-charge distribution is computed using a combination of the particle-in-cell method (PICM) with the area-weighted algorithm (AWA) for allocation of the charges to the mesh [13]. The cells are defined by the same mesh of step \( H \), which is used for the solution of the boundary-value problem. The electron beam is represented by a finite number of \( N \) rays, each carrying a fraction \( I_i \) of the total beam current.

\[ I_i = \sum_{i=1}^{N} I_i. \]  

For each time step \( \Delta t \) inside a given cell, the charge

\[ Q_d = I_i \Delta t, \]  
(12)

is deposited to the four adjacent vertices of the cell (mesh nodes) according to the AWA, which for a square mesh reduces to

\[ Q_a = \left(1 - \frac{H_x}{H}\right) \left(1 - \frac{H_y}{H}\right) Q_d, \]  
(13)
where \( H_R \) and \( H_Z \) are the distances along \( R \) and \( Z \) axes between the corresponding \( n \)-th node and the position of the particle in the middle of the ray region considered. The space-charge density in the node with radial coordinate \( R_n \) is then given by

\[
\rho_n = \frac{Q_n}{2\pi H^2 R_n}.
\]  

(14)

Allocating \( \rho_n \) and summing up the contribution of all rays the charge density in each node is obtained.

Although in the MIG for medium power submillimeter gyrotrons the emitter operates in a temperature limited mode, we incorporate the Langmuir's theory in our physical model in order to extend the applicability of the code to MIGs used in high power gyrotrons, where the emitter may operate in a space-charge limited mode. According to this approach, we assume that the region near the emitter can be divided into a number of small virtual diodes in which the current is governed by potential distribution and initial velocities of the thermoelectrons. It is also assumed that in each virtual diode the Langmuir's theory holds and the technique described in [9] is applied for computation of the extracted currents.

The major steps of the algorithm realized in the GUN-MIG software package are:

(i) Input of initial data and parameters as well as logical keys specifying the content of numerical experiments and output information.

(ii) Analysis of the geometry of the gun, boundary conditions and generation of an appropriate mesh system. Once the geometry has been analyzed, the program proceeds with the main iterative portion of the code responsible for a self-consistent solution of the problem:

(iii) Solution of the Laplace's equation by finite-difference method (FDM) with successive overrelaxation (SOR) on a sequence of three grids of increasing fineness.

(iv) Computation of the extracted current density in each element of the annular emitting ring.

(v) A finite number of electron trajectories, each associated with a definite current density is then traced through the obtained electric field by the fourth-order Runge-Kutta method.

(vi) Computation of the space charge density distribution by the PIC method using the Area Weighted Algorithm (AWA) for allocating the charge to the mesh points.
(vii) Solution of the Poisson's equation with the space charge distribution obtained in the previous step.

Steps (iv)-(vii) are then repeated until a self-consistent solution is obtained.

(viii) The final step of the numerical experiment includes processing of obtained data as well as output of the results. This includes computation of statistical estimates (mean values, standard deviation, relative standard deviation) characterizing the velocity distributions of electrons in the beam. Although the present version of the package traces electron trajectories only in the gun, some extrapolations (based on the adiabatic theory) for the resonant cavity are also being calculated on this stage.

GUN-MIG is written in FORTRAN-77 and is operational on various IBM-compatible PCs. It consists of a set of computational modules and postprocessor program intended for systematization and visualization of the results of numerical experiments.

3. Illustrative examples

Basic requirements which must be satisfied by the MIG can be formulated as follows:

(i) Formation of high quality beam which can be transported without wall losses. This requirement is of primary importance for high frequency, harmonic gyrotrons which have narrow cavities. Narrow cavities are used in order to get a good mode separation and in such a way to operate the gyrotron in many single modes at the fundamentals, second and third harmonics. For example, the diameter of the cavity in Gyrotron FU IV [14] from Gyrotron FU Series developed in the Fukui University is only 3.23 mm. The use of such narrow cavities however increases the demands concerning the alignment of the system as well as makes the transport of the beam and minimization of the wall losses more difficult. The misalignment of the beam decreases the energy transfer rate and increases the starting current for gyrotron operation.

(ii) Electrode configuration (e.g. cathode angle) which permits operation over a wide current range without any significant changes of other beam parameters.
(iii) Tunability of the velocity ratio $\alpha = v_\perp / v_\parallel$ (where $v_\perp$ and $v_\parallel$ are the perpendicular and the parallel velocity components) by adjusting the anode voltage in a wide magnetic field intensity range. It should be noted that the efficiency of the gyrotron depends strongly on $\alpha$ which is limited by beam mirroring.

(iv) Modulation of the anode voltage in order to modulate the velocity distribution of the electron beam, which in turn modulates gyrotron output [15].

(v) Frequency step switching (due to switching of operating cavity mode) and frequency modulation by modulating the energy of the electron beam [16].

(vi) Precise control of the injection point of the electron beam in the cavity. The electron beam should enter the cavity at such a radius as to minimize the starting current.

(vii) The MIG has to form a beam with minimal velocity spread because the increased velocity spread leads to electron beam mirroring at lower values of $\alpha$. Even though the total velocity is the same for all electrons in the beam because they pass the same accelerating potential difference, there is a spread in their axial and transverse velocities. Electrons originating from different parts of the emitter arrive at the cavity with different velocities and make different contribution to the conversion of the energy in the gyrotron. It should be noted that the velocity spreads of $v_\perp$ and $v_\parallel$ are correlated and the only spread in total electron energy is that due to the space-charge potential depression in the drifting region.

To achieve these demands, the electron gun should be carefully designed. The choice of the configuration of the gun is often a compromise between many contradictory requirements. A typical axisymmetric MIG of triode type is shown on Fig. 1. The first electrode is a cathode with an emitting ring (thermionic emitter). The next two electrodes with positive potentials with respect to the cathode are the anode and the main body of the gyrotron (which may be considered as a “second anode”). Thus, the acceleration of the electrons and formation of a helical annular beam takes place in a combination of axial and radial electric fields with a magnetic field. Also shown on Fig. 1 are some electron trajectories of the beam formed at constant magnetic field $B_1 = 0.222$ T. The total beam current in this case is $I_\parallel = 1.0$ A.
Fig. 1 Configuration of the MIG-A and electron trajectories (scales along R and Z axes are different)

Usually the number of plotted trajectories (15 in our illustration) is much smaller than the total number of trajectories used for computation of the space charge distribution. In this particular example the emitting ring was divided into 100 annular regions of equal area and, correspondingly, 100 trajectories originating from them were used for space charge allocation. It should be noted that electron orbits are three-dimensional \((R = R(Z), \theta = \theta(Z))\) and do not lie in the plane of the plot. Strictly speaking, on Fig. 1 only the dependence \(R = R(Z)\) for different trajectories is portrayed actually, rather than the trajectories themselves. Although incomplete, such simple and conventional representation gives certain notion about the configuration of the beam.

For more detailed insight into the beam structure however it is helpful to know how the individual trajectories and the beam as a whole evolve along the longitudinal axis. As an example, a projection of a selected ray on the \(X-Y\) plane is shown on Fig. 2. On Fig. 3 the change of the azimuthal angle as the electron follows its path along the orbit of Fig. 2 is shown.
Fig. 2 Projection of the orbit N8 on the X-Y plane (dashed line represents the emitting ring)

Fig. 3 Azimuthal angle vs. longitudinal coordinate for the electron orbit of Fig.2
Two other characteristics of the beam are presented on Fig. 4 and Fig. 5. The first one shows the relation between the initial \( R_i \) and the final \( R_o \) radial co-ordinates of the electrons in the final cross-section.

![Graph showing the relation between final and initial radii](image)

**Fig. 4** Final vs. initial radius of electron trajectories in the exit plane of the MIG-A

![Graph showing the velocity ratio vs. radial coordinate](image)

**Fig. 5** Velocity ratio vs. radial coordinate for the selected orbits
of the gun. The second one gives the velocity ratio ($\alpha$) vs. $R_0$ for the
same transverse cross-section of the beam.

Fig. 6 shows the emittance pattern of the beam in the final cross-
section. Such plots can be obtained in different transverse cross-sections
and allow one to trace the changes which the beam structure undergoes.
In our illustrative example all electron trajectories start with zero initial
velocities and hence the emittance of the beam is zero as well. The
emittance diagrams are both excellent and very sensitive tools for
studding of initial velocities effects and their influence on the beam
quality. We hope to extend the theory of phase-space analysis [9] to
helical electron beams in order to develop the necessary computational
modules and techniques for phase analysis of electron beams generated
in MIGs.

\[ R \ (\text{cm}) \]

\[ R = dR/dZ \ (\text{rad}) \]

Fig.6 Emittance pattern of the beam at the exit of MIG-A

On Fig. 7 and Fig. 8 two other MIGs are shown just to illustrate the
dependence of both topology and structure of generated beams on the
geometrical configuration of the electrodes. For comparison, some of the
parameters that characterize the geometry of the investigated guns are
presented in Table I. Among them, the angle of the emitting surface is
particularly important. The increase of the angle of inclination of the cathode emitting surface leads to a decrease of the transverse initial velocity.

**Fig. 7** Configuration of the MIG-B and electron trajectories.

**Fig. 8** Configuration of the MIG-C and electron trajectories.
velocities of electrons and to a formation of beams which structure is closer to the laminar one.

Table I Geometrical parameters of the analyzed MIGs
(θ - angle of inclination of the emitting surface;
S₀ - area of the emitting surface; L₀ - length of the generatrix of the emitting surface; Rₑ₀ - average radius of the emitter; Dₑ₀ - cathode-anode distance)

<table>
<thead>
<tr>
<th>GUN</th>
<th>θ (deg)</th>
<th>S₀ (mm²)</th>
<th>L₀ (mm)</th>
<th>Rₑ₀ (mm)</th>
<th>Dₑ₀/Rₑ₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIG-A</td>
<td>17.10</td>
<td>88.62</td>
<td>3.40</td>
<td>4.15</td>
<td>1.31</td>
</tr>
<tr>
<td>MIG-B</td>
<td>15.52</td>
<td>52.21</td>
<td>1.87</td>
<td>4.45</td>
<td>0.87</td>
</tr>
<tr>
<td>MIG-C</td>
<td>24.62</td>
<td>102.33</td>
<td>3.30</td>
<td>4.94</td>
<td>1.21</td>
</tr>
</tbody>
</table>

On Fig. 9, the dependence of the velocity ratio on the magnetic field in MIG-A is shown. The dominant tendency is a decrease of the velocity.
ratio with the decrease of the E/B ratio on the cathode which affects the initial transverse velocities of electrons. Analogous dependence for MIG-A, MIG-B and MIG-C at \( U_a = 15 \text{ kV} \) is presented on Fig. 10. The fluctuations which can be observed on these plots are due to the fact that on the initial parts of their paths the electrons traverse different regions, in which, generally speaking, the E/B ratio is different. As a result the velocity ratio does not depend monotonically on B.

![Graph showing velocity ratio vs. magnetic field for MIG-A, MIG-B and MIG-C at anode potential \( U_a = -25 \text{ kV} \).]

Fig. 10 Velocity ratio vs. magnetic field for MIG-A, MIG-B and MIG-C at anode potential \( U_a = -25 \text{ kV} \).

4. Conclusion

In this paper we have presented the current version of GUN-MIG - a new problem oriented software package for computer simulation of beam formation in MIGs. Its structure permits addition of new modules and allows for ease of code maintenance and modification. Some further extensions to both the model and the code are planned by introducing
the analysis of beam transport to the cavity in convergent magnetic field. Our final objective is the development of both multicriteria optimization methods and software for CAD of high performance MIGs.

Acknowledgments

This work was performed as a part of an ongoing joint research project between the Laboratory of Physical Problems of Electron Beam Technologies at the Institute of Electronics of the Bulgarian Academy of Sciences (Sofia, Bulgaria) and Laboratory for Application of Superconducting Magnet at the Faculty of Engineering in the Fukui University (Fukui, Japan).

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